THE EFFECT OF LIQUID FLOW SWIRL ON THE INTENSIFICATION OF CONVECTION HEAT TRANSFER IN A CIRCULAR CYLINDRICAL TUBE

V. P. Kharitonov

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We present the results from an analytical study of the distribution of velocities and heat-transfer characteristics for a single case of vortex flow in a compressible ideal liquid, under the condition that the vortex/velocity vector ratio is constant. We examine the effect of flow swirling on the intensification of heat transfer under the conditions of the internal problem.

A method of improving the heat-engineering characteristics of industrial liquid heat exchanges involves the utilization of a swirling flow. Technologically, this is accomplished rather simply by employing various types of external or internal swirlers. However, the complex nature of the convection heat transfer which proceeds under conditions of mutual application of forced motions compels us to resort primarily to experimental methods of investigation. From the experimental aspect, this problem – whether for one- or two-phase flows – has therefore been studied rather thoroughly [1-4]. However, the development of theoretical concepts and the determination of quantitative values for the heat flows in a stream of a liquid or gas is, as yet, far from concluded.

In this connection, we should cite references [4-6], in which the authors provide a theoretical scheme for the calculation of the friction and heat-transfer characteristics for the developed turbulent motion of a flow with constant swirling.

We propose a theoretical scheme for the study of the velocity fields and heat-transfer characteristics under the condition that the vortex/velocity vector ratio is constant. Here particular attention is devoted to examining the effect of liquid flow swirl on the intensification of the convection heat transfer in the case of forced flow convection in tubes. The studies are carried out in a range of variation in Re from $1 \cdot 10^2$ to $1 \cdot 10^5$, i.e., to the limit of completely developed turbulent motion.

1. In cylindrical coordinates, let us examine the steady-state two-parameter motion of a swirled liquid flow in a circular cylindrical tube of radius r whose walls are kept at a temperature T_1. The liquid entering the tube at a specified pressure distribution exhibits the temperature $T_0(y)$. We will assume that at the inlet section the longitudinal velocity component u_x is constant, and that the flow swirl velocity u_θ is proportional to the radius.

Let the liquid flow in the tube satisfy the Gromeki–Beltrami condition [7]:

$$\frac{\Omega_x}{u_x} = \frac{\Omega_y}{u_y} = \frac{\Omega_\theta}{u_\theta} = \frac{\lambda}{2}. \tag{1}$$

Proceeding from the general gas dynamic equations for an ideal liquid, taken in the Lamb–Gromeki form, with condition (1) we obtain the required system of equations:

$$\frac{\partial}{\partial x}(y\rho u_x) + \frac{\partial}{\partial y}(y\rho u_y) = 0, \tag{2}$$

$$\rho c_p\left(u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{1}{y}\frac{\partial}{\partial y}\left(ky \frac{\partial T}{\partial y}\right), \tag{3}$$

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$$q = -k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y}\right), \tag{4}$$

$$p = \rho RT, \tag{5}$$

$$\frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{6}$$

$$\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{7}$$

$$\frac{\partial u_\theta}{\partial x} = -\lambda u_y, \tag{8}$$

$$\frac{1}{y}\frac{\partial}{\partial y}(y u_\theta) = \lambda u_x, \tag{9}$$

$$\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = \lambda u_\theta. \tag{10}$$

It has been taken into consideration in Eqs. (2)-(10) that the flow parameters are independent of θ, i.e., they are functions exclusively of the coordinates x and y. The x-axis coincides with the tube axis and is directed in the direction of the flow, while the y-axis is directed along the tube radius. In Eq. (2) we have neglected the dissipated heat [8], while in Eqs. (6) and (7), in the light of the forced motion, we have neglected the mass forces. Equations (8)-(10) represent nothing other than condition (1), written in projections onto the indicated coordinate axes.

We see from Eqs. (3)-(5) that to find the temperature fields and the coefficients of thermal conductivity, and then the heat-flow coefficient, we must first determine the velocity field, the density, and the pressure.

2. Let us substitute the values of u_x and u_y from relationships (8) and (9) into (10). Here, after simple transformations, to determine u_θ we derive the equation

$$\frac{\partial^2 u_\theta}{\partial x^2} + \frac{\partial^2 u_\theta}{\partial y^2} + \frac{1}{y}\frac{\partial u_\theta}{\partial y} + \left(\lambda^2 - \frac{1}{y^2}\right) u_\theta = 0. \tag{11}$$

We will seek its solution for the boundary conditions

$$u_\theta(0, y) = \omega_0 y, \quad u_\theta(x, 0) = 0, \quad u_\theta(x, r) = \omega_0 r, \tag{12}$$

as well as under the condition that the function u_θ is bounded and that the value of u_x is constant at the inlet cross section.

Let us introduce the new function

$$\Phi(x, y) = u_\theta(xy) - \omega_0 y \tag{13}$$

and we will substitute its value into Eq. (11) and into the boundary conditions (12), following which we have

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{1}{y}\frac{\partial \Phi}{\partial y} + \left(\lambda^2 - \frac{1}{y^2}\right)\Phi = -\omega_0 \lambda^2 y, \tag{14}$$

$$\Phi(0, y) = 0, \quad \Phi(x, 0) = 0, \quad \Phi(x, r) = 0. \tag{15}$$

Applying the method separating variables, we will seek the solution to Eq. (14) under conditions (15) in the form of a series which can be differentiated twice term by term with respect to x and y,

$$\Phi(x, y) = \sum_{n=1}^{\infty} \Phi_n(x) J_1(\mu_n y), \tag{16}$$

where $J_1(\mu_n y)$ is a Bessel function of the first kind. Here the eigenfunctions $J_1(\mu_n y)$ have been derived from the solution of the uniform equation corresponding to Eq. (14), and from the boundary conditions with respect to the variable y. The eigennumbers $\mu_n = \varkappa_n / r$ in this case are determined in terms of the roots of the transcendental equation $J_1(\varkappa_n) = 0$.

Substitution of series (16) into Eq. (14) yields

$$\sum_{n=1}^{\infty}\left[\Phi_n''(x)-(\mu_n^2-\lambda^2)\Phi_n(x)\right]J_1(\mu_n y)=-\omega_0\lambda^2 y. \tag{17}$$

Hence we have

$$\Phi_n''(x)-(\mu_n^2-\lambda^2)\Phi_n(x)=C_n, \tag{18}$$

where C_n are the Fourier–Bessel coefficients for the function $(-\omega_0\lambda^2 y)$, i.e.,

$$C_n=\frac{\int\limits_0^r -\omega_0\lambda^2 y^2 J_1(\mu_n y)\,dy}{\int\limits_0^r y\left[J_1(\mu_n y)\right]^2 dy}=\frac{2\omega_0 r\lambda^2}{\varkappa_n J_0(\varkappa_n)}.$$

The solution to Eq. (18) for the boundary conditions (15) with respect to the variable x is found in the form

$$\Phi_n(x)=\frac{2\omega_0 r\lambda^2\left[\exp\left(-\sqrt{\mu_n^2-\lambda^2}\,x\right)-1\right]}{\varkappa_n(\mu_n^2-\lambda^2)J_0(\varkappa_n)}. \tag{19}$$

Thus the sought solution to Eq. (11) under the boundary conditions (12), with consideration of relationships (13), (16), and (19), is written as

$$u_\theta(x,\ y)=\omega_0 y+\sum_{n=1}^{\infty}L_n\left[\exp(-b_n x)-1\right]J_1\left(\frac{\varkappa_n y}{r}\right), \tag{20}$$

where

$$L_n=\frac{2\omega_0\lambda^2 r^3}{\varkappa_n J_0(\varkappa_n)(\varkappa_n^2-\lambda^2 r^2)},\quad b_n=\frac{\sqrt{\varkappa_n^2-\lambda^2 r^2}}{r}.$$

Series (20) and the series derived after twice differentiating that series term by term with respect to x and y converge uniformly. As a proof it is sufficient to note that they are, respectively, majorized by absolutely converging series composed of the absolute values of their coefficients $\sum\limits_{n=1}^{\infty}|L_n|$, $\sum\limits_{n=1}^{\infty}|2\omega_0\lambda^2 r/\varkappa_n J_0(\varkappa_n)|$, etc.

With consideration of (20), from relationships (8) and (9) we will have

$$u_x=\frac{2\omega_0}{\lambda}+\frac{1}{\lambda}\sum_{n=1}^{\infty}\frac{\varkappa_n L_n}{r}\left[\exp(-b_n x)-1\right]J_0\left(\frac{\varkappa_n y}{r}\right), \tag{21}$$

$$u_y=\sum_{n=1}^{\infty}\frac{b_n L_n}{\lambda}\left[\exp(-b_n x)\right]J_1\left(\frac{\varkappa_n y}{r}\right). \tag{22}$$

Equation (22) shows that the resulting solution of the problem does not contradict the condition of non-passage at the tube wall, i.e., $u_y = 0$ when $y = r$, while (21) for $x = 0$ and given values of u_x and u_y makes it possible to determine the coefficient of flow intensity λ.

3. For certain values of u_x and u_y the change in density can now be found from Eq. (2) which, in the light of conditions (8) and (9), assumes the form

$$u_x\frac{\partial\rho}{\partial x}+u_y\frac{\partial\rho}{\partial y}=0. \tag{23}$$

The auxiliary system of ordinary differential equations is written [9] in the form

$$\frac{dx}{u_x}=\frac{dy}{u_y}=\frac{d\rho}{0}. \tag{24}$$

Its first integrals are

$$\rho = C_1, \quad -\omega_0 y^2 + \sum_{n=1}^{\infty} L_n (1 - \exp(-b_n x))\, y J_1\left(\frac{\varkappa_n y}{r}\right) = C_2.$$

The following function will then serve as the general solution for the original equation (23), in a form solved for ρ:

$$\rho = F\left[-\omega_0 y^2 + \sum_{n=1}^{\infty} L_n (1 - \exp(-b_n x))\, y J_1\left(\frac{\varkappa_n y}{r}\right)\right], \tag{25}$$

where F is an arbitrary differentiable function.

The solution which satisfies the initial condition $\rho(0, y) = f(y)$, as follows from the specified distribution of pressure and temperature in the inlet cross section, is written in the form

$$\rho = f\left(\sqrt{y^2 - \frac{1}{\omega_0} \sum_{n=1}^{\infty} L_n [1 - \exp(-b_n x)]\, y J_1\left(\frac{\varkappa_n y}{r}\right)}\right). \tag{26}$$

The pressure distribution in the flow will now determine Eqs. (6) and (7), from which we have

$$p = -\left[\int \rho \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) dx + \int \rho \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right) dy\right] + C, \tag{27}$$

where C is the integration constant which we can assume to be known for a specified value of the temperature T_1, with consideration of relationship (26) for y = r, i.e.,

$$p = -\int_y^r \left[\rho \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) dx + \rho \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right) dy\right] + (\rho R T_1)_{y=r}. \tag{28}$$

The temperature field in the liquid flow, satisfying the boundary conditions $T(0, y) = T_0(y)$, and $T(x, r) = T_1$, will be determined by relationship (5). Equation (3) will then determine the coefficient of thermal conductivity, while Eq. (4) will give us the heat flow. When k = const, the relationship for the heat flow assumes the form

$$q = -\frac{\rho c_p \left(u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y}\right)}{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{1}{y}\frac{\partial T}{\partial y}} \left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y}\right), \tag{29}$$

where the parameters u_x, u_y, ρ, p, and T are, respectively, determined by relationships (21), (22), (26), (27), and (5).

We note that to determine the temperature, instead of (5) we can use the equation of state which would describe the physically most realistic process.

We see from the above that a swirling flow is characterized primarily by the profile of the tangential velocity component. Depending on the changes of the latter, we determine the thermal characteristics, the flow structure, and the extent of the swirling. In turn, the value of u_θ is found to be directly dependent on the angular velocity ω_0 or, in other words, dependent on the angle at which the flow of the working medium is supplied. It is for this reason that we will subsequently dwell on an investigation of the interrelationship between the intensification of heat transfer and the angular velocity ω_0.

An examination of relationship (29) shows that the heat flow increases with a rise in ω_0. Intensively swirled flows of the medium produce the greatest effect of increasing heat transfer in this case, and this intensifying effect is all the more perceptible at the inlet segment of the tube.

Without dwelling here on an examination of the possible special cases of flow and heat transfer, in conclusion we will offer a clarifying example on the basis of our conclusions for the results of the calculations with formula (29) for air at the following parameter values: $T_0 = 400°C$, $T_1 = 350°C$, $c_p = 1.07 \cdot 10^3$ J/kg·deg, $\mu = 2.18 \cdot 10^{-5}$ kg/m·sec, r = 0.05 m, R = 8.317 J/deg·mole for x > 0, y = 0.99 r, and $\rho_0 = 1/RT_0\,(10^4 - 0.1y)$, ω_0 varying from 1 to 600 1/sec. We found the value of $\varkappa_n$ and the necessary values of the functions J_0 and J_1 from the tables in [10].

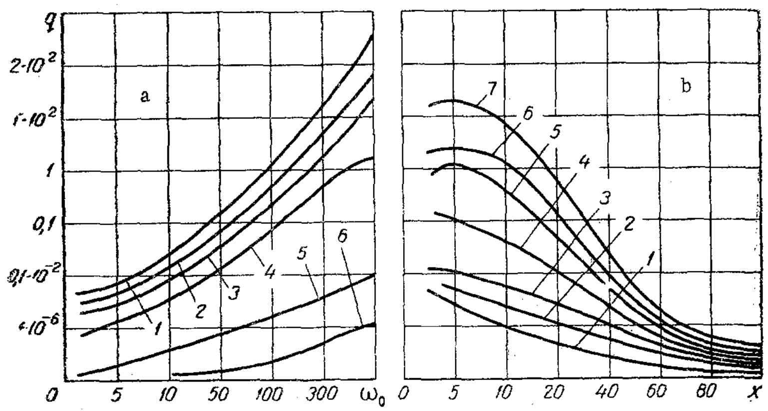

Fig. 1. A change in q, in cal/cm · sec, as a function of ω_0, in 1 /sec, at various points in the tube: a – 1) for x = 3; 2) 6; 3) 9; 4) 20; 5) 50; 6) 70; and as a function of x, in cm, along the walls of the tube: b – 1) for ω_0 = 1; 2) 4; 3) 8; 4) 40; 5) 80; 6) 240; 7) 400.

Because the temperature and velocity fields were formulated gradually, in calculating the results shown in Fig. 1 we failed to take into consideration the inlet segment which is equal to r/2. In finding the flow parameters we took into consideration two terms of the series. The calculation that were carried out with consideration of three and four terms of the series showed that the difference between the successive approximations is very small (agreement up to four significant figures). This indicates the rapid convergence of the series. All of the determined integrals with respect to the variable y were found numerically in the calculations on the basis of the general Simpson formula.

A study of formula (29) and our calculations show that the qualitative aspect of the problem with regard to the change in the heat flow is in good agreement with the experimental formula in [1, 2] and with the generalized theoretical formula (4.1) in reference [11]. However, the absence of special experimental data defining the heat flow under specified conditions prevents us from undertaking a direct quantitative evaluation.

4. It follows from an examination of these results that significant intensification effect is reduced slightly with an increase in the Reynolds number.

The studies of the heat-transfer results will probably be more valid for the core of the flow, since it is in this region that the viscosity effect is less perceptible. However, it is not surprising that the form of the motion here was found to resemble the motion of a liquid devoid of viscosity. As had been demonstrated by Milovich [12], this is because the imposition of condition (1) on the flow automatically introduces an actual viscosity effect.

The theoretical formulas derived above enable us most simply to establish the quantitative relationships governing the change in heat flow from the kind and temperature of the liquid, from the velocity and regime of the flow, from the diameter and length of the tube, etc. The excellent agreement in the qualitative picture of the change in the heat flow can be regarded as confirmation of the fact that the analysis procedure had been properly chosen.

NOTATION

Ω_x, Ω_y, Ω_θ are the vortex components;
u_x, u_y, u_θ are the velocity vector components;
λ = const is the flow intensity coefficient;
T is the temperature;
p is the pressure;
ρ is the density;
q is the specific heat flow;
k is the coefficient of thermal conductivity;
c_p is the specific heat capacity;
ω_0 = const is the angular velocity of the flow;
R is the gas constant;
$u^2 = u_x^2 + u_y^2 + u_\theta^2$.

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